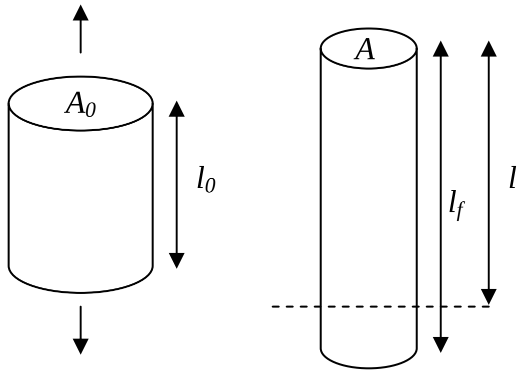
Introduction to Mechanics

True stress and true strain:

Engg. stress is given by $\sigma_{engg} = \frac{Load}{A_0}$

Engg. strain is given by $\varepsilon_{engg.} = \frac{\Delta l}{l_0}$



In real practice, the area of cross-section changes as one applies load to strain the specimen. Considering no change in volume:

$$A_0 l_0 = Al$$

True stress is given by

$$\sigma_{true} = \frac{Load}{A} = \frac{Load}{A_0\left(\frac{l_0}{l}\right)}$$

$$\sigma_{true} = \frac{Load}{A_0} \times \frac{l}{l_0}$$

$$\sigma_{true} = \sigma_{engg.}(1 + \varepsilon_{engg.})$$

True strain is given by

$$d\varepsilon_{true} = \frac{dl}{l}$$

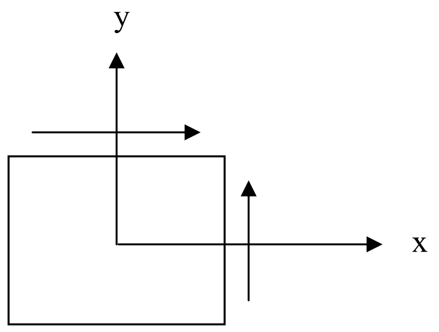
$$\int d\varepsilon_{true} = \int_{l_0}^{l_f} \frac{dl}{l}$$

$$\varepsilon_{true} = \ln \frac{l_f}{l_0}$$

$$\varepsilon_{true} = \ln\left(\frac{l_0 + \Delta l}{l_0}\right) = \ln\left(1 + \frac{\Delta l}{l_0}\right)$$

$$\varepsilon_{true} = \ln\left(1 + \varepsilon_{engg.}\right)$$

Hooke's Law for plane stress:



Stresses are applied to the material in figure above and strains are studied. Considering that the material remains elastic by the application of the stresses along the axes, we derive the equation for strains in the material.
First, let us consider the normal strain due to application of stress.

$$\sigma_x = E\varepsilon_x$$

and

$$\sigma_y = E\varepsilon_y$$

but due to Poisson's contraction one obtains the value of strain as –

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right]$$

similarly,

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\sigma_x\right]$$

and

$$\varepsilon_z = -\frac{\nu}{E}\left[\sigma_x + \sigma_y\right]$$

Using Cramer's Rule:

$$\sigma_x = \frac{\begin{vmatrix} 1 & E\varepsilon_y \\ -\nu & E\varepsilon_x \end{vmatrix}}{\begin{vmatrix} 1 & -\nu \\ -\nu & 1 \end{vmatrix}}$$

$$\sigma_x = \frac{E\varepsilon_x + \nu E\varepsilon_y}{1-\nu^2}$$

$$\sigma_x = \frac{E}{1-\nu^2}\left[\varepsilon_x + \nu\varepsilon_y\right]$$

Similarly, one can obtain the stress in y direction as –

$$\sigma_y = \frac{E}{1-\nu^2}\left[\varepsilon_y + \nu\varepsilon_x\right]$$

In addition to the above stresses we have shear stress in terms of the shear strain:

$$\tau_{xy} = G\gamma_{xy}$$

When no stress is applied in the z direction, we get what is collectively called ‘Hooke’s Law of Plane Stress’. They contain three material constants (E, G and ν), but only two are independent because of the relationship

$$G = \frac{E}{2[1+\nu]}$$

Hooke’s Law for triaxial stress (stresses act in all three directions):

If the material follows Hooke’s law, we can obtain the relation between normal stress and normal strain by using the same procedure as above.

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\left[\sigma_y + \sigma_z\right]$$
$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\left[\sigma_z + \sigma_x\right]$$
$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}\left[\sigma_x + \sigma_y\right]$$

Rearranging the above set of equations, one can find the principal stresses as –

$$\sigma_x = \frac{\begin{vmatrix} E\varepsilon_x & -\nu & -\nu \\ E\varepsilon_y & 1 & -\nu \\ E\varepsilon_z & -\nu & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{vmatrix}}$$

Similarly, one can obtain the other principal stresses as well.

Thus, solution of the above involves algebraic manipulations as under.

$$\sigma_x = \frac{E\varepsilon_x(1-\nu^2) - E\varepsilon_y(-\nu-\nu^2) + E\varepsilon_y(\nu^2+\nu)}{(1+\nu^2)+\nu(-\nu-\nu^2)-\nu(\nu^2+\nu)}$$

$$\sigma_x = \frac{E\left[\varepsilon_x(1+\nu)(1-\nu)+\varepsilon_y\nu(1+\nu)+\varepsilon_y\nu(1+\nu)\right]}{(1+\nu)(1-\nu)-2\nu^2(1+\nu)}$$

$$\sigma_x = \frac{E\left[(1-\nu)\varepsilon_x+\nu(\varepsilon_y+\varepsilon_z)\right]}{(1+\nu)(1-2\nu)}$$

Similarly, one can obtain expressions for the stresses in the other directions.

$$\sigma_y = \frac{E\left[(1-\nu)\varepsilon_y+\nu(\varepsilon_z+\varepsilon_x)\right]}{(1+\nu)(1-2\nu)}$$

$$\sigma_z = \frac{E\left[(1-\nu)\varepsilon_z+\nu(\varepsilon_x+\varepsilon_y)\right]}{(1+\nu)(1-2\nu)}$$

These equations represent Hooke's Law for triaxial stress.